3.6/27)

$x^2+y^2=1$

a) $(1/\sqrt{2}, 1/\sqrt{2})$

b) $(1/\sqrt{2}, -1/\sqrt{2})$

find slope of tangent line
→ solve for y & diff
→ implicit diff.

i) solve for y

$x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$

iA) $y=+\sqrt{1-x^2}$

$+(1-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$\frac{dy}{dx}=\frac{-2x}{2\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$

a) at $P:\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\left.\frac{dy}{dx}\right|_P=\frac{-\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}}$

$=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\frac{1}{2}}}=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}}=-1$

iB) $y=-\sqrt{1-x^2}$

$\frac{dy}{dx}=-\frac{1(-2x)}{2\sqrt{1-x^2}}$

$\frac{dy}{dx}=\frac{x}{\sqrt{1-x^2}}$

does not apply

ii) implicit diff

$x^2+y^2=1$

$2x+2y\left(\frac{dy}{dx}\right)=0$

$\frac{dy}{dx}=\frac{-x}{y}$

$\frac{dy}{dx}=\frac{-x}{y}$

$\frac{dy}{dx}=\frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}=-1$

3.6/37)

$a^4 - t^4 = 6a^2t$ find $\frac{da}{dt}$ → differentiating with respect to t.

a is a function of t.
i.e. $a = a(t)$

$$4a^3\frac{da}{dt} - 4t^3 = 6\frac{d}{dt}(a^2t)$$

$$4a^3\frac{da}{dt} - 4t^3 = 6\left(\left(2a\frac{da}{dt}\right)(t) + (a^2)(1)\right)$$

$$4a^3\frac{da}{dt} - 4t^3 = 12at\frac{da}{dt} + 6a^2$$

$$4a^3\frac{da}{dt} - 12at\frac{da}{dt} = 6a^2 + 4t^3$$

$$\frac{da}{dt}(4a^3 - 12at) = 6a^2 + 4t^3$$

$$\frac{da}{dt} = \frac{6a^2+4t^3}{4a^3-12at} = \frac{3a^2+4t^3}{2a(a^2-3t)}$$

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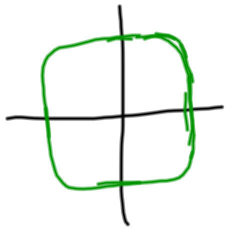
29) $x^4+y^4=16$; $(1, \sqrt[4]{15})$



d.i. $4x^3+4y^3\frac{dy}{dx}=0$

$$\frac{dy}{dx}=\frac{-x^3}{y^3} \quad \text{so} \quad \left.\frac{dy}{dx}\right|_{P=(1,\sqrt[4]{15})} = \frac{-1}{15^{3/4}}$$

3.8) differentials and local linear approximation

MAJOR MAJOR IDEA.

If an equation of a line with slope m through (x_0, y_0) is:

$$y - y_0 = m(x - x_0)$$

AND if the slope of a tangent line is given by the value of the derivative,

Then

$$y - y_0 = f'(x_0)(x - x_0)$$ is the equation of the tangent line.

and rewriting slightly...

$$T(x) = f(x_0) + f'(x_0)(x - x_0)$$

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MAJOR IDEA continued - -

$$T(x) = f(x_0) + f'(x_0)(x - x_0)$$

The tangent line,
for values "close to" the point of tangency
MAY BE USED as an approximation for the function

LOCAL LINEAR APPROXIMATION

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Use a local linear approximation to estimate $\sqrt{7}$.

function: $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\quad \boxed{\sqrt{7} = 2.645}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

point of tangency: $(9, \sqrt{9}) = (9, 3)$

what we want: $\sqrt{7} = (x_0 = 7,\ f(x_0) = \sqrt{7})$

$$T(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$T(x) = 3 + \frac{1}{2\sqrt{9}}(x - 9)$$

Using $(9,3)$ as pt of tangency

$$\text{sqrt}(x) \approx T(x) = 3 + \frac{1}{6}(x-9)$$

$$\sqrt{7} \approx 3 + \frac{1}{6}(7-9) = 3 + \frac{1}{6}(-2) = 3 - \frac{1}{3} = \frac{8}{3} \approx 2.667$$

$T(x) = 9 + \frac{9}{18} = 9\frac{1}{2}$

$9 + \frac{1}{2(9)}(90 - 81)$

$T(x) = 10 + \frac{1}{20}(x - 100)$

$10 + \frac{1}{20}(90 - 100)$

$10 + \left(-\frac{1}{2}\right)$ 9.5

$\sqrt{90} = 9.4868\ldots$

approx $e^{\left(\frac{1}{3}\right)}$ $\approx 1.39....$

function: $y = e^x$

$y' = e^x$

pt of tangency: $(0, 1)$

$T(x) = 1 + (1)(x - 0)$

$e^{\frac{1}{3}} \approx 1 + 1\left(\frac{1}{3} - 0\right) = \frac{4}{3}$